Stability of acoustic streaming flows in plane channels

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We study the stability of acoustic streaming flows of normal fluids induced by a small-amplitude surface acoustic wave propagating along the walls of a confined parallel-plane channel or slab in the incompressible flow regime. The secondary flows derived are of negligible effect to the stability characteristic after comparing with the primary flow. The governing equation which was derived by considering the weakly nonlinear coupling between the wavy wall and viscous fluid is obtained and then the eigenvalue problem is solved by a numerical code together with the associated dynamic and kinematic conditions. The value of the critical Reynolds number was found to be near 4873 which is smaller than the case 5772 for conventional pressure-driven flows.

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I. INTRODUCTION

Surface acoustic wave (SAW) is an interesting and important subject in physics as well as in many other scientific applications [1,2]. One example is the characterization of physical and chemical properties of complicated materials. The other example is related to the possible means of pumping molecules and atoms near or along surfaces under the perturbing influence of SAW [3]. In fact, with the latter, it has been reported that people can make microflow systems that could circulate heat-transfer fluids over silicon chips, reconstitute dried drugs, and possibly synthesize chemicals from liquid and solid constituents [4].

While Moroney *et al.* used 4.7 MHz ultrasonic Lamb waves in a 4-μm-thick composite membrane of silicon nitride (2 μm) and piezoelectric zinc oxide (1 μm) to induce pumping of water (inside a 250-μm-deep well), Nguyen *et al.* applied the same technique almost at the same frequency range to a microsystem of 15-μm and 500-μm channel for pumping air as well as water [4]. The key point is that since the acoustic (A_0-mode flexural plate) wave velocity is much lower than the sound velocity in typical fluids, the microstructure (system) acts as a nearly lossless acoustic waveguide when in contact with a fluid. This acoustically driven micropump offers the advantages of low operating voltages and gentle pumping with no valves involved. Thus, they could be also implemented in a wide range of physical, chemical, and biological applications (e.g., a few DNA separation or monitoring) [5].

Acoustic or steady streaming flow, or, the mean flow induced by the interaction of surface waves and elastic boundaries has been intensively studied since late 1800s [3,4,6–9]. The original or relevant approach (considering the liquid) could even be dated to early 1830s (by Faraday [8(a)]). Note that the theoretical treatment of streaming in an acoustical boundary layer at a plane wall was first conducted successfully by Rayleigh [8(c)] but that was for standing waves only. Faraday's experiments also were on standing waves. The crucial step, from the point of view of both applications and theory, was made in treating progressive (i.e., traveling) waves. This was done originally by Longuet-Higgins [8(c)]. He treated both the no-slip case and the zero-stress case. Later on, we also noticed that the first experimental confirmation of the streaming in traveling waves was found by Longuet-Higgins (1960) for the solid wall, and later by the same author for a fluid-free surface [9].

Recently, due to the rapid progresses in SAW sensing [10], there are renewed interests in this research field. Especially for gases, preliminary theoretical results in a free molecular and continuum regime showed some agreement with previous approaches (e.g., the latter regime compared with liquid) [3,4]. According to the calculation in the nearly continuum regime, the flow velocity of the gas in a microtube is proportional to the second power of the SAW displacement velocity (product of the SAW amplitude and circular frequency) and this has also been experimentally confirmed (please see Ref. [11] for the cited references therein).

We know, however, that most of the sensors, e.g., those operated in microelectromechanical system (MEMS) applications, were performed in wide dynamic range and high sensitivity. Microchannels built in the microstructure of MEMS are easily subjected to environment noises, such as oscillations or vibrations, externally excited traveling waves, etc. The stability characteristic for the acoustic streaming flow is thus of considerable importance to the flow control or mixing in microdomain since the bulk (cross-section) size of the microchannel is at most a few tens of $O(\mu\text{m})$ and the wall of the microchannel is almost in submicrons or lesser.

As we know, it is necessary to obtain curves of the neutral stability boundary from the stability equation (say, the well-known Orr-Sommerfeld equation; the basic flow could be a shear flow or plane Poiseuille flow) and associated boundary conditions for the description of the hydrodynamical transition to turbulence in normal fluids. In this paper, we shall first derive the relevant governing equations and boundary conditions. The primary (steady) and secondary (time-averaged) flows thus obtained will be examined considering

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the relative order of magnitude. As the latter is much smaller compared with the former when we take into account the case of the small-amplitude (acoustic) wave, we only investigate the stability problem related to those basic flows [induced by the steady streaming, i.e., Eq. (6) derived in Sec. II] with dynamic and kinematic boundary conditions [e.g., cf. parameter K_0 in Eq. (9)] linked to the perturbed waves by using the verified numerical code (via the spectral method [12,13]) [14]. We found that once the parameter K_0 [comes from the boundary conditions Eq. (9)] becomes 0.1, the critical Reynolds number becomes 4873 which is less than the conventional case ($\approx$5772, obtained by Orszag using the spectral method [12]).

II. FORMULATIONS

We consider a plane channel of uniform thickness filled with a homogeneous viscous fluid. The walls of the channel on which traveling sinusoidal waves of small amplitude a are not absolutely rigid are imposed. The vertical displacements of the upper and lower walls ($y=d$ and $-d$) are thus presumed to be η and $-\eta$, respectively, where $\eta=a\cos[2\pi(x-ct)/\lambda_a]$, λ_a being the wave length and c the wave speed. x and y are Cartesian coordinates, with x measured in the direction of (SA) wave propagation and y measured in the direction normal to the mean position of the walls.

It would be expedient to simplify those equations by introducing dimensionless variables. We have a characteristic velocity c and three characteristic lengths a, λ_a, and d. The following variables based on c and d could thus be introduced:

$$x'=\frac{x}{d},\quad y'=\frac{y}{d},\quad u'=\frac{u}{c},\quad v'=\frac{v}{c},\quad \eta'=\frac{\eta}{d},$$

$$\psi'=\frac{\psi}{c\,d},\quad t'=\frac{c\,t}{d},\quad p'=\frac{p}{\rho c^2}.$$

The amplitude ratio ϵ, the wave number α, and the Reynolds number Re are defined by

$$\epsilon=\frac{a}{d},\quad \alpha=\frac{2\pi d}{\lambda_a},\quad \mathrm{Re}=\frac{c\,d}{\nu}.$$

We shall seek a solution in the form of a series in the parameter ϵ:

$$\psi=\psi_0+\epsilon\psi_1+\epsilon^2\psi_2+\cdots,$$

$$\frac{\partial p}{\partial x}=\left(\frac{\partial p}{\partial x}\right)_0+\epsilon\left(\frac{\partial p}{\partial x}\right)_1+\epsilon^2\left(\frac{\partial p}{\partial x}\right)_2+\cdots,$$

with

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}.$$

In this study, we assume that the Mach number is rather small: Ma$\ll$1 (consistent with previous experiments [4] or similar approaches [11]), and then the adopted equations are the incompressible Navier-Stokes equations which are associated with the no-slip boundary conditions along the wavy walls. The two-dimensional (x and y) momentum equations and the equation of continuity could be in terms of the stream function ψ if the pressure (p) term is eliminated. The final governing equation is

$$\frac{\partial}{\partial t}\nabla^2\psi+\psi_y\nabla^2\psi_x-\psi_x\nabla^2\psi_y=\frac{1}{\mathrm{Re}}\nabla^4\psi,\quad \nabla\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, \tag{1}$$

and subscripts indicate the partial differentiation. Thus, we have

$$\frac{\partial}{\partial t}\nabla^2\psi_0+\psi_{0y}\nabla^2\psi_{0x}-\psi_{0x}\nabla^2\psi_{0y}=\frac{1}{\mathrm{Re}}\nabla^4\psi_0, \tag{2}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_1+\psi_{0y}\nabla^2\psi_{1x}+\psi_{1y}\nabla^2\psi_{0x}-\psi_{0x}\nabla^2\psi_{1y}-\psi_{1x}\nabla^2\psi_{0y}=\frac{1}{\mathrm{Re}}\nabla^4\psi_1, \tag{3}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_2+\psi_{0y}\nabla^2\psi_{2x}+\psi_{1y}\nabla^2\psi_{1x}+\psi_{2y}\nabla^2\psi_{0x}\psi_{0x}-\nabla^2\psi_{2y}-\psi_{1x}\nabla^2\psi_{1y}-\psi_{2x}\nabla^2\psi_{0y}=\frac{1}{\mathrm{Re}}\nabla^4\psi_2, \tag{4}$$

and other higher order forms. The fluid is subjected to boundary (dynamic and kinematic) conditions imposed by the symmetric motion of the walls: $u=0$, $v=\pm\partial\eta/\partial t$ at $y=\pm(1+\eta)$. The boundary conditions may be expanded in powers of η and then ϵ:

$$\psi_{0y}|_1+\epsilon[\cos\alpha(x-t)\,\psi_{0yy}|_1+\psi_{1y}|_1]+\epsilon^2\left[\frac{\psi_{0yyy}|_1}{2}\cos^2\alpha(x-t)+\psi_{2y}|_1+\cos\alpha(x-t)\,\psi_{1yy}|_1\right]+\cdots=0,$$

$$\psi_{0x}|_1+\epsilon[\cos\alpha(x-t)\,\psi_{0xy}|_1+\psi_{1x}|_1]+\epsilon^2\left[\frac{\psi_{0xyy}|_1}{2}\cos^2\alpha(x-t)+\cos\alpha(x-t)\,\psi_{1xy}|_1+\psi_{2x}|_1\right]+\cdots=-\epsilon\alpha\sin\alpha(x-t). \tag{5}$$

The above equations, together with the condition of symmetry and a uniform pressure gradient in the x direction, $(\partial p/\partial x)_0=$constant, yield

$$\psi_0=K_0\left(y-\frac{y^3}{3}\right),\quad K_0=\frac{\mathrm{Re}}{2}\left(-\frac{\partial p}{\partial x}\right)_0, \tag{6}$$

$$\psi_1=\frac{1}{2}\{\phi(y)e^{i\alpha(x-t)}+\phi^*(y)e^{-i\alpha(x-t)}\}, \tag{7}$$

where the asterisk denotes the complex conjugate. A substitution of ψ_1 into Eq. (3) yields

$$\left\{\frac{d^2}{dy^2}-\alpha^2+i\alpha\mathrm{Re}[1-K_0(1-y^2)]\right\}\left(\frac{d^2}{dy^2}-\alpha^2\right)\phi - 2i\alpha K_0\mathrm{Re}\phi=0. \tag{8}$$

The boundary conditions are

$$\phi_y(\pm 1)=2K_0, \quad \phi(\pm 1)=\pm 1. \tag{9}$$

Similarly, with

$$\psi_2=\frac{1}{2}\{D(y)+E(y)e^{i2\alpha(x-t)}+E^*(y)e^{-i2\alpha(x-t)}\}, \tag{10}$$

we have

$$D_{yyyy}=-\frac{i\alpha\mathrm{Re}}{2}(\phi\phi_{yy}^*-\phi^*\phi_{yy})_y, \tag{11}$$

$$\left[\frac{d^2}{dy^2}-(4\alpha^2-2i\alpha\mathrm{Re})\right]\left(\frac{d^2}{dy^2}-4\alpha^2\right)E = i2\alpha\mathrm{Re}K_0(1-y^2)\left(\frac{d^2}{dy^2}-4\alpha^2\right)E+i4\alpha K_0\mathrm{Re}E + \frac{i\alpha\mathrm{Re}}{2}(\phi_y\phi_{yy}-\phi\phi_{yyy})=0, \tag{12}$$

and the boundary conditions

$$D_y(\pm 1)+\frac{1}{2}[\phi_{yy}(\pm 1)+\phi_{yy}^*(\pm 1)]-2K_0=0,$$

$$E_y(\pm 1)+\frac{1}{2}\phi_{yy}(\pm 1)-\frac{K_0}{2}=0.$$

A. Secondary effects due to free pumping

To simplify the approach and obtain preliminary analytical solutions of the above complicated equations and boundary conditions, we first consider the case in which $(\partial p/\partial x)_0$ vanishes or $K_0=\psi_0=0$. This corresponds to the secondary effect when we neglect the primary basic flow. Hence Eqs. (9) and (10) become

$$\left(\frac{d^2}{dy^2}-\alpha^2\right)\left(\frac{d^2}{dy^2}-\overline{\alpha^2}\right)\phi=0, \quad \overline{\alpha^2}=\alpha^2-i\alpha\mathrm{Re}, \tag{13}$$

$$\phi_y(\pm 1)=0, \quad \phi(\pm 1)=\pm 1. \tag{14}$$

After lengthy algebraic manipulations, we obtain

$$\phi=c_0e^{\alpha y}+c_1e^{-\alpha y}+c_2e^{\overline{\alpha}y}+c_3e^{-\overline{\alpha}y},$$

where $c_0=(A+A_0)/\det$, $c_1=-(B+B_0)/\det$, $c_2=(C+C_0)/\det$, $c_3=-(T+T_0)/\det$;

$$\det=Ae^{\alpha}-Be^{-\alpha}+Ce^{\overline{\alpha}}-Te^{-\overline{\alpha}},$$

$$A=e^{\alpha}\overline{\alpha^2}(r^2e^{-2\overline{\alpha}}-s^2e^{2\overline{\alpha}})-2\alpha\overline{\alpha}e^{-\alpha}w + \alpha\overline{\alpha}e^{\alpha}z(e^{-2\overline{\alpha}}r+e^{2\overline{\alpha}}s),$$

$$A_0=e^{-\alpha}\overline{\alpha^2}(r^2e^{-2\overline{\alpha}}-s^2e^{2\overline{\alpha}})+2\alpha\overline{\alpha}e^{\alpha}z - \alpha\overline{\alpha}e^{-\alpha}w(e^{2\overline{\alpha}}s+e^{-2\overline{\alpha}}r),$$

$$B=e^{-\alpha}\overline{\alpha^2}(r^2e^{-2\overline{\alpha}}-s^2e^{2\overline{\alpha}})+2\alpha\overline{\alpha}e^{\alpha}z - \alpha\overline{\alpha}e^{-\alpha}w(e^{-2\overline{\alpha}}r+e^{2\overline{\alpha}}s),$$

$$B_0=e^{\alpha}\overline{\alpha^2}(r^2e^{-2\overline{\alpha}}-s^2e^{2\overline{\alpha}})-2\alpha\overline{\alpha}e^{-\alpha}w + \alpha\overline{\alpha}e^{\alpha}z(e^{-2\overline{\alpha}}r+e^{2\overline{\alpha}}s),$$

$$C=e^{-\alpha}\alpha\overline{\alpha}(wse^{\overline{\alpha}-\alpha}-rze^{\alpha-\overline{\alpha}})-\alpha e^{2\alpha+\overline{\alpha}}z(\alpha z-\overline{\alpha}s) + \alpha e^{-\alpha}w(\alpha e^{\overline{\alpha}-\alpha}w-\overline{\alpha}e^{\alpha-\overline{\alpha}}r),$$

$$C_0=e^{\alpha}\alpha\overline{\alpha}(wse^{\overline{\alpha}-\alpha}-rze^{\alpha-\overline{\alpha}})-\alpha z(z\alpha e^{2\alpha-\overline{\alpha}}-\overline{\alpha}e^{\overline{\alpha}}s) + \alpha w(\alpha e^{-(\overline{\alpha}+2\alpha)}w-\overline{\alpha}e^{-(2\alpha+\overline{\alpha})}r),$$

$$T=e^{-\alpha}\alpha\overline{\alpha}(zse^{\overline{\alpha}+\alpha}-rwe^{-(\alpha+\overline{\alpha})})-\alpha\overline{\alpha}(e^{2\alpha-\overline{\alpha}}zr-e^{\overline{\alpha}}ws) + \alpha^2e^{-\alpha}(-e^{2\alpha}z^2+e^{-2\alpha}w^2),$$

$$T_0=e^{\alpha}\alpha\overline{\alpha}(zse^{\overline{\alpha}+\alpha}-rwe^{-(\alpha+\overline{\alpha})})-\alpha\overline{\alpha}(e^{-\overline{\alpha}}zr-e^{\overline{\alpha}-2\alpha}ws) + \alpha^2e^{\alpha}(-e^{2\alpha}z^2+e^{-2\alpha}w^2),$$

with $r=s=w=z=1$.

To obtain a simple solution which relates to the mean flow so long as only terms of $O(\epsilon^2)$ are concerned, we see that if every term in the x-momentum equation is averaged over an interval of time equal to the period of oscillation [11], we obtain for our solution, as given by above equations, the mean pressure gradient

$$\overline{\frac{\partial p}{\partial x}}=\epsilon^2\overline{\left(\frac{\partial p}{\partial x}\right)_2} = \epsilon^2\left[\frac{D_{yyy}}{2\mathrm{Re}}+\frac{i\mathrm{Re}}{4}(\phi\phi_{yy}^*-\phi^*\phi_{yy})\right]+O(\epsilon^3) = \epsilon^2\frac{a_0}{\mathrm{Re}}+O(\epsilon^3), \tag{15}$$

where a_0 is the integration constant for the integration of Eq. (12). Now, from Eq. (14), we have

$$D_y(\pm 1)=-\frac{1}{2}[\phi_{yy}(\pm 1)+\phi_{yy}^*(\pm 1)],$$

where $D_y(y)=a_0y^2+a_1y+a_2+\mathcal{C}(y)$, and from Eq. (12),

$$\mathcal{C}(y)=\frac{\alpha^2\mathrm{Re}^2}{2}\Bigg[\frac{c_0c_2^*}{g_1^2}e^{(\alpha+\overline{\alpha}^*)y}+\frac{c_0^*c_2}{g_2^2}e^{(\alpha+\overline{\alpha})y}$$
$$+\frac{c_0c_3^*}{g_3^2}e^{(\alpha-\overline{\alpha}^*)y}+\frac{c_0^*c_3}{g_4^2}e^{(\alpha-\overline{\alpha})y}+\frac{c_1c_2^*}{g_3^2}e^{(\overline{\alpha}^*-\alpha)y}$$
$$+\frac{c_1^*c_2}{g_4^2}e^{(\overline{\alpha}-\alpha)y}+\frac{c_1c_3^*}{g_1^2}e^{-(\overline{\alpha}^*+\alpha)y}+\frac{c_1^*c_3}{g_2^2}e^{-(\overline{\alpha}+\alpha)y}$$
$$+\frac{c_2c_3^*}{g_5^2}e^{(\overline{\alpha}-\overline{\alpha}^*)y}+\frac{c_2^*c_3}{g_5^2}e^{(\overline{\alpha}^*-\overline{\alpha})y}+2\frac{c_2c_2^*}{g_6^2}e^{(\overline{\alpha}^*+\overline{\alpha})y}$$
$$+2\frac{c_3c_3^*}{g_6^2}e^{-(\overline{\alpha}^*+\overline{\alpha})y}\Bigg],$$

with $g_1=\alpha+\overline{\alpha}^*$, $g_2=\alpha+\overline{\alpha}$, $g_3=\alpha-\overline{\alpha}^*$, $g_4=\alpha-\overline{\alpha}$, $g_5=\overline{\alpha}-\overline{\alpha}^*$, and $g_6=\overline{\alpha}+\overline{\alpha}^*$. In practical applications we must determine a_0 from considerations of conditions at the ends of the channel. a_1 equals to zero because of the symmetry of boundary conditions.

Once a_0 is specified [11], our solution for the mean speed (averaged over time) of secondary flow is

$$\overline{U}=\epsilon^2\frac{D_y}{2}=\frac{\epsilon^2}{2}\{\mathcal{C}(y)-\mathcal{C}(1)+R_0+a_0[y^2-1]\}, \quad (16)$$

where $R_0=-[\phi_{yy}(1)+\phi_{yy}^*(1)]/2$, which has a numerical value about 3 for a wide range of α and Re.

For small amplitude waves, say, $\epsilon=0.01$ or smaller, $\overline{U}$ is, at most, of the order of magnitude $O(0.001)$ which is rather small compared to the primary flow obtained from ψ_0 [cf. Eq. (6) for $K_0\neq=0$]. For fit our present interests here, we shall only consider the stability of basic flows obtained from ψ_0 [$K_0\neq 0$, with boundary conditions due to the acoustic streaming, say Eq. (9)].

B. Primary flow stability

To obtain the stability characteristics for acoustic streaming flows by using verified codes developed before [14] (calculating the Orr-Sommerfeld spectra), we need to transform Eq. (8) into the Orr-Sommerfeld form by rescaling and redimensionalization of physical parameters and variables mentioned before (e.g., the careful selection of K_0 and c). Thus, we have the linearized disturbance equation

$$(D^2-\alpha^2)(D^2-\alpha^2)\phi = i\alpha\mathrm{Re}[(\overline{u}-C)(D^2-\alpha^2)\phi-(D^2\overline{u})\phi], \quad (17)$$

where $D=d/dy$, $C=C_r+iC_i$. C_r is the ratio between the velocity of the propagating perturbation wave and the characteristic velocity, and C_i is the amplification factor. As for the temporal stability problem, in which the growth or decay of a disturbance in time is considered, we treat the (complex) wave speed C as the eigenvalue parameter of the problem. The mean (basic) velocity profile is given by [cf. Eq. (6)]

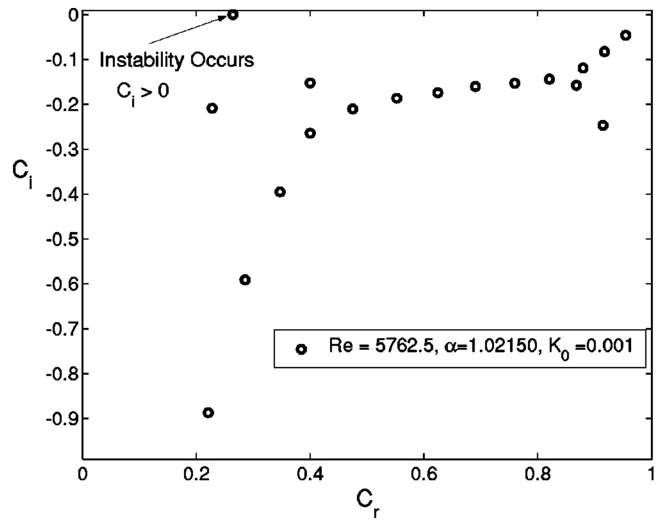

FIG. 1. Illustration of the temporal spectra (C_r, C_i) for disturbance waves due to acoustic streaming ($K_0=0.001$) effects. The upper branch: Re$=5762.5$ which is near $\mathrm{Re}_{cr}=5762.3$ for corresponding $\alpha\approx 1.02$.

$$\overline{u}=1-y^2 \quad (18)$$

for $-1\leqslant y\leqslant 1$. Boundary and interface conditions for ϕ or $D\phi$ are already defined in Eq. (9) and are not the same as previous approaches [12,14] [$\phi(\pm 1)=D\phi(\pm 1)=0$ therein].

The eigenvalue problem raised above could then be solved by using the verified code [14], which follows or adopts the spectral method [13] based on the Chebyshev-polynomial-expansion approach, after the equation and boundary conditions are discretized. The algebraic or matrix equation (cf. Refs. [11,13]) is similar to that expressed in [12] (by Orszag), except the dynamic and kinematic boundary conditions which become

$$\sum_{\substack{n\equiv 0 (\mathrm{mod}\ 2)\\ n=0}}^{N} a_n=1, \qquad \sum_{\substack{n\equiv 0 (\mathrm{mod}\ 2)\\ n=0}}^{N} n^2a_n=2K_0, \quad (19)$$

$$\sum_{\substack{n\equiv 1 (\mathrm{mod}\ 2)\\ n=1}}^{N} a_n=-1, \qquad \sum_{\substack{n\equiv 1 (\mathrm{mod}\ 2)\\ n=1}}^{N} n^2a_n=2K_0. \quad (20)$$

Here we adapt Osborne's algorithm [15] to first precondition these complex matrices via rescaling, i.e., by certain diagonal similarity transformations of the matrix (errors are in terms of the Euclidean norm of the matrix) designed to reduce its norm [14]. The details of this algorithm could be traced in Ref. [14]. The form of the reduced matrix is then upper Hessenberg. We then perform the stabilized LR transformations for these matrices to get the eigenvalues $C=C_r+iC_i$ (please see also Ref. [14] for the details).

III. RESULTS AND DISCUSSION

The preliminary verified results of this numerical code had been reported [14,16] for the cases of no-slip and slip

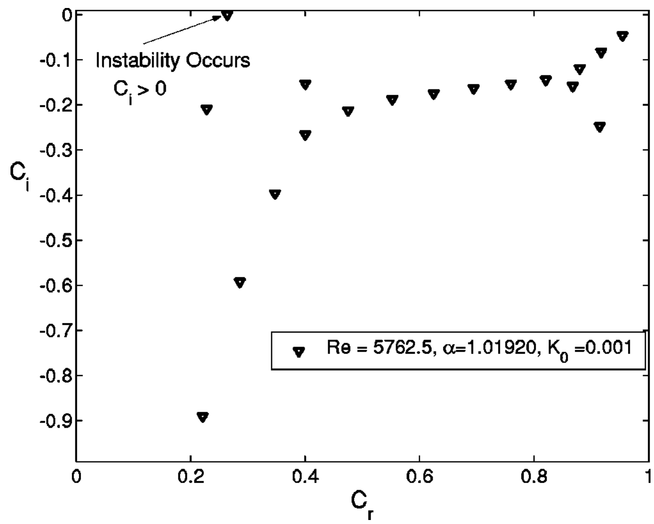

FIG. 2. Illustration of the temporal spectra (C_r, C_i) for disturbance waves due to acoustic streaming $(K_0 = 0.001)$ effects. The lower branch: Re$=5762.5$ which is near Re$_{cr}=5762.3$ for corresponding $\alpha \approx 1.02$. All the units as shown in Figs. 1 and 2 (and 3) are dimensionless.

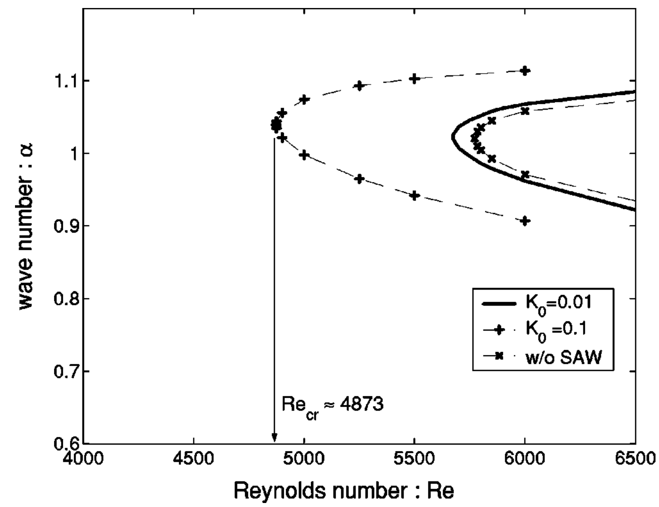

FIG. 3. Comparison of the effects due to acoustic streaming: $K_0 = 0.01$ and 0.1 on the neutral stability boundary of the plane Poiseuille flow. Re$_{cr}$ (the critical Reynolds number of the flow) $\approx$ 5673.5 and 4873 for different cases.

boundary conditions in comparison with the bench-mark results of Orszag [12]. For example, we obtained the same spectra (e.g., case of Re$=10\,000$ and $\alpha=1$) and critical Reynolds number (Re$_{cr}\sim 5772.2$) for the test case: the plane Poiseuille flow which Orszag obtained from CDC 7600 [12]. We then perform intensive calculations considering present objectives. The searching of Re$_{cr}$ and corresponding α for our interests $(K_0 \neq =0)$ here, however, is time consuming and depends on the experiences.

We subsequently calculated those spectra for $K_0 \geqslant 0$ (effects due to acoustic streaming) with the associated dynamic and/or kinematic boundary (interface) conditions by carefully adjusting the Reynolds number (Re) and the wave number α. With preliminary calculations, we first found that as $K_0 = 0.001$, Re$_{cr}$ decreases to around 5762.3. The temporary spectra (C_r, C_i) for both branches near this value were plotted into Figs. 1 and 2 for the direct illustration of effects due to acoustic streaming. Note that all the units shown in Figs. 1 and 2, (and 3) are using quantities that are dimensionless here. The instability mode triggers for the mode $C_r \approx 0.2642$ once Re$=5762.5$ and $\alpha = 1.0215$ for the upper branch. Similar spectra for the lower branch (Re$=5762.5$ and $\alpha = 1.0192$) are shown in Fig. 2 which indicates that the instability mode triggers for the mode $C_r \approx 0.2639$.

We can also obtain the neutral boundary curves for specific Re and α with $K_0 = 0.01, 0.1$ and plot them Fig. 2. For comparison with previous static-wall case [11], we put previous results (without K_0) into Fig. 3. For those flows with Re and α falling within the specific curve or the neutral boundary (Re$\geqslant$Re$_{cr}$ and α bounded in between), they are unstable. That is to say, any small disturbance will amplify in a finite time and/or in the downstream when Re is larger than the critical ones (Re$_{cr}$ with respect to the specific range of α) which are fixed upon the curves. We have roughly Re$_{cr}$ $=5673.5$ and 4873 for $K_0 = 0.01$ and 0.1 cases, respectively. These are smaller than the critical Reynolds number of conventional cases (static wall: 5772) for the same no-slip flows. Note that, to use Eq. (17) with suitable transformations, once the (SA) wave speed c is large, we must select (corresponding) small K_0 (less than one) [16]!

To conclude in brief, we can observe that the instability of the flow due to acoustic streaming in plane channels occurs much earlier (Re$_{cr}$ becomes 4873 for $K_0 = 0.1$), which is not favorable for the flow control in most applications. This latter observation might be interpreted as due to the weakly nonlinear coupling between the wall boundary and the inertia of the acoustic streaming flow. For the optimal flow-control usage in common scientific applications, either the range of (SA) wave numbers relevant to the noise wave [17] or the Reynolds number of the basic flows must be carefully selected.

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